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Quantum Poincaré group related to the κ-Poincaré algebra

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Abstract. The classical r-matrix implied by the quantum κ-Poincaré algebra of Lukierski, Nowicki and Ruegg is used to generate a Poisson structure on the Poincaré group. A quantum deformation of the Poincaré group (on the Hopf *-algebra level) is obtained by a trivial quantization.

1. Introduction

The theory of quantum groups [1–4] offers a framework for new types of physical symmetries. From this point of view, a study of quantum deformations of the Poincaré group is one of the first steps to test the (intriguing, but still problematic) applicability of quantum groups to fundamental symmetries.

The 'phase diagram' of possible quantum deformations of the Poincaré group in four dimensions is not yet known (it is known in two dimensions [5], and also in the case of the Lorentz group [6]). Although the existence of several families of deformations is expected (we already know a six-parameter family of so-called 'soft deformations' [7, 8]), the problem has not been sufficiently investigated and, strictly speaking, no such deformation can be found in the literature (in [9] the Poincaré group enlarged by dilatations is studied).

On the level of quantized universal enveloping algebras, there is only one example in the literature, known as the κ-Poincaré algebra [10, 11], which was obtained by a contraction of $\mathcal{U}_q(o(3,2))$ (an example with dilatations is given in [12]).

In this paper we construct an example of a quantum deformation of the Poincaré group (on the Hopf *-algebra level). Our procedure consists of two steps. In section 2 we construct an example of a Poisson Poincaré group using a particular classical r-matrix. In section 3 we replace the Poisson brackets by commutators (this trivial method works in this case!) and obtain a Hopf *-algebra. There is also a quantum Minkowski space on which the quantum Poincaré group acts.

The source (the whole information) for this deformation is the classical r-matrix, a rather simple object (equation (7) below). We have deduced this r-matrix from the cocommutator (or 'cobracket', see [1]) implied by the κ-Poincaré algebra (cf section 4). Therefore we expect our example of a quantum Poincaré group to play the role of the dual of the κ-Poincaré algebra. We did not attempt to make this point more rigorous (this would require long and tedious calculations). Instead, we present the dual Poisson group of our Poisson Poincaré group (section 5). It turns out that this dual Poisson group is described by the same equations as the κ-Poincaré algebra, with commutators replaced by Poisson brackets (multiplied by the imaginary unit). This shows that the κ-Poincaré algebra is obtained by a trivial quantization from this dual Poisson group and supports our conjecture about quantum duality.

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The theory of Poisson Lie groups (which we call Poisson groups) was developed in [1, 13–15]). The classification of Poisson structures on a given Lie group is the same as the classification of quantum deformations of the group in all known cases (e.g. see a recent comparison [16] for the Lorentz group). We hope to return to the classification of all Poisson structures on the Poincaré group in another paper.

2. An example of the Poisson Poincaré group

We denote by G the Poincaré group in four dimensions, identified here as the group of matrices

$$g = (g^a{}_b)_{a,b=0,\ldots,4} = \begin{pmatrix} \Lambda & v \\ 0 & 1 \end{pmatrix}$$

where $\Lambda = (\Lambda^\mu_\nu)$ belongs to the Lorentz group and $v = (v^\mu) \in \mathbb{R}^4$ $(\mu, \nu = 0, 1, 2, 3)$. Commutators of standard generators M_i, L_i $(i = 1, 2, 3)$, P_μ $(\mu = 0, 1, 2, 3)$ of the Lie algebra $\mathfrak{g}$ of G are given by

$$[M_i, M_j] = \varepsilon_{ijk} M_k \qquad [P_\mu, P_\nu] = 0 \tag{1}$$

$$[L_i, M_j] = \varepsilon_{ijk} L_k \tag{2}$$

$$[M_i, P_j] = \varepsilon_{ijk} P_k \qquad [M_i, P_0] = 0 \tag{3}$$

$$[L_i, P_j] = \delta_{ij} P_0 \qquad [L_i, P_0] = P_i \tag{4}$$

$$[L_i, L_j] = -\varepsilon_{ijk} M_k \tag{5}$$

(using the summation convention), where ε_{ijk} $(i, j, k = 1, 2, 3)$ is the totally anti-symmetric symbol such that $\varepsilon_{123} = 1$. If we denote by e_a $(a = 0, 1, 2, 3, 4)$ the standard basis in $\mathbb{R}^5$ and by $e_a{}^b := e_a \otimes e^b$ the standard basis in $End(\mathbb{R}^5)$ (e^b is the dual basis of e_a), then

$$M_i = \varepsilon_{ijk} e_k{}^j \qquad L_i = e_0{}^i + e_i{}^0 \qquad P_\mu = e_\mu{}^4 \tag{6}$$

where $i, j, k = 1, 2, 3$ and $\mu = 0, 1, 2, 3$.

Now consider $r \in \bigwedge^2 \mathfrak{g}$ given as follows (cf section 4):

$$r = h \sum_{k=1}^{3} L_k \wedge P_k \tag{7}$$

(h is a real deformation parameter). A calculation of the Schouten bracket [1, 13–15] of r with itself yields

$$[r, r] = 2hr \wedge P_0 - h^2 \sum_{i,j,k=1}^{3} \varepsilon_{ijk} P_i \wedge P_j \wedge M_k. \tag{8}$$

It is not difficult to see that $[r, r]$ is invariant, hence r defines a structure of a Poisson Lie group on G. The Poisson bivector π on $G \subset End(\mathbb{R}^5)$ is given by $\pi(g) = gr - rg$, i.e.

$$\pi(g) = h \sum_{\mu,\nu=0}^{3} \sum_{k=1}^{3} \left\{ (g^\mu{}_0 e_\mu{}^k + g^\mu{}_k e_\mu{}^0) \wedge g^\nu{}_k e_\nu{}^4 - (g^k{}_\mu e_0{}^\mu + g^0{}_\mu e_k{}^\mu) \wedge g^4{}_\nu e_k{}^\nu \right\} \tag{9}$$

(we have used the equations $ge_a{}^b = \sum_c g^c{}_a e_c{}^b$ and $e_a{}^b g = \sum_c g^b{}_c e_a{}^c$ for the left and right translations, applied to $r = \sum_{k=1}^3 (e_0{}^k + e_k{}^0) \wedge e_k{}^4$). Taking into account

$$g^4{}_0 = g^4{}_1 = g^4{}_2 = g^4{}_3 = 0 \qquad g^4{}_4 = 1 \qquad g^\mu{}_\nu = \Lambda^\mu{}_\nu \qquad g^\mu{}_4 = v^\mu \tag{10}$$

with $\mu, \nu \leqslant 3$, we obtain

$$\begin{aligned}\pi(g) = {} & h\sum_{k=1}^{3} v^k e_k{}^4 \wedge e_0{}^4 + h\sum_{\mu=0}^{3}\sum_{k=1}^{3}(\Lambda^\mu{}_0 e_\mu{}^k + \Lambda^\mu{}_k e_\mu{}^0)\Lambda^0{}_k \wedge e_0{}^4 \\ & + h\sum_{n=1}^{3}\Bigg\{-(\Lambda^n{}_0 e_0{}^0 + \Lambda^0{}_0 e_n{}^0) - \sum_{k=1}^{3}(\Lambda^n{}_k e_0{}^k + \Lambda^0{}_k e_n{}^k) \\ & + \sum_{\mu=0}^{3}\sum_{k=1}^{3}(\Lambda^\mu{}_0 e_\mu{}^k + \Lambda^\mu{}_k e_\mu{}^0)\Lambda^n{}_k\Bigg\} \wedge e_n{}^4.\end{aligned}$$

This enables us to calculate the Poisson brackets of the coordinate functions on G

$$\{\Lambda^\mu{}_\nu, \Lambda^\lambda{}_\rho\} = 0 \tag{11}$$

$$\{v^k, v^j\} = 0 \tag{12}$$

$$\{v^k, v^0\} = hv^k \tag{13}$$

$$\{\Lambda^\mu{}_\nu, v^0\} = h(\Lambda^\mu{}_0\Lambda^0{}_\nu - \delta^{\mu 0}\delta^0{}_\nu) \tag{14}$$

$$\{\Lambda^m{}_0, v^j\} = h(\delta^{mj} - \delta^{mj}\Lambda^0{}_0 + \Lambda^m{}_0\Lambda^j{}_0) \tag{15}$$

$$\{\Lambda^0{}_0, v^j\} = h(\Lambda^0{}_0\Lambda^j{}_0 - \Lambda^j{}_0) \tag{16}$$

$$\{\Lambda^0{}_n, v^j\} = h(\Lambda^0{}_0\Lambda^j{}_n - \Lambda^j{}_n) \tag{17}$$

$$\{\Lambda^m{}_n, v^j\} = h(-\delta^{mj}\Lambda^0{}_n + \Lambda^m{}_0\Lambda^j{}_n) \tag{18}$$

$(\mu, \nu, \lambda, \rho = 0, 1, 2, 3;\ k, j = 1, 2, 3)$. Equations (14)–(18) may be also written in a compact form

$$\{\Lambda^\mu{}_\nu, v^\rho\} = h\{(\Lambda^\mu{}_0 - \delta^\mu{}_0)\Lambda^\rho{}_\nu + \eta^{\mu\rho}(\Lambda^0{}_\nu - \delta^0{}_\nu)\} \tag{19}$$

where $\eta = diag(1, -1, -1, -1)$. Also (12), (13) may be written as one equation

$$\{v^\alpha, v^\beta\} = h(v^\alpha\delta^{\beta 0} - v^\beta\delta^{\alpha 0}). \tag{20}$$

Let $\mathfrak{h}$ be the Lie algebra defined by these relations ($\mathfrak{h}$ is spanned by v^α). From (19), to each $X \in \mathfrak{h}$ there corresponds a vector field $\hat{X}$ on the Lorentz group L such that

$$\hat{X}(\Lambda^\mu{}_\nu) = \{X, \Lambda^\mu{}_\nu\}. \tag{21}$$

By the Jacobi identity, $X \mapsto \hat{X}$ defines an (infinitesimal) action of $\mathfrak{h}$ on L. It is clear that the Poisson structure on G is encoded in this action. The Poisson structure is in fact the semidirect Poisson structure $L \rtimes \mathfrak{h}^*$ (cf [17]).

3. The quantum Poincaré group

Let $\mathcal{A}$ be the universal *-algebra with unity, generated by self-adjoint elements $\Lambda^\mu{}_\nu$, v^μ ($\mu, \nu = 0, 1, 2, 3$), subject to the relations

$$[\Lambda^\mu{}_\nu, \Lambda^\lambda{}_\rho] = 0 \tag{22}$$

$$[v^\alpha, v^\beta] = \mathrm{i}h(v^\alpha \delta^{\beta 0} - v^\beta \delta^{\alpha 0}) \tag{23}$$

$$[\Lambda^\mu{}_\nu, v^\rho] = \mathrm{i}h\{(\Lambda^\mu{}_0 - \delta^\mu{}_0)\Lambda^\rho{}_\nu + \eta^{\mu\rho}(\Lambda^0{}_\nu - \delta^0{}_\nu)\} \tag{24}$$

$$\Lambda^\mu{}_\nu \Lambda^\lambda{}_\rho \eta^{\nu\rho} = \eta^{\mu\lambda}. \tag{25}$$

These relations arise from (11), (19) and (20) by replacing Poisson brackets by commutators divided by the imaginary unit. This procedure is unambiguous: there is no ordering ambiguity when 'quantizing' the right-hand side of (19), due to the commutativity in (11). One can show that the spaces of polynomials of a given degree in variables $\Lambda^\mu{}_\nu$, v^μ have the same dimension as in the non-deformed case ($h = 0$), i.e. the algebra $\mathcal{A}$ has a 'proper size' and can be regarded as a deformation of the algebra of polynomials on the Poincaré group. *Moreover, since the standard comultiplication Δ is compatible with Poisson brackets* (11), (19) and (20), it is also compatible with the defining relations (22)–(25). Indeed, the expressions for Poisson brackets coincide with the expressions for commutators and the calculation of the compatibility conditions look identical in both cases. We conclude that the above relations together with the standard comultiplication

$$\Delta\Lambda^\mu{}_\nu = \Lambda^\mu{}_\rho \otimes \Lambda^\rho{}_\nu \qquad \Delta v^\mu = \Lambda^\mu{}_\nu \otimes v^\nu + v^\mu \otimes I \tag{26}$$

define a Hopf *-algebra. The antipode is given by the familiar equation

$$S(\Lambda^\mu{}_\nu) = (\Lambda^{-1})^\mu{}_\nu \qquad S(v^\mu) = -(\Lambda^{-1})^\mu{}_\nu v^\nu \tag{27}$$

where $(\Lambda^{-1})^\mu{}_\nu = \eta^{\mu\rho}\Lambda^\lambda{}_\rho \eta_{\nu\lambda}$ is the usual inverse matrix of Λ.

One can check (see also proposition 1 below) that relations (22)–(25) can be written in the following R-matrix form

$$R^{ab}{}_{cd} g^c{}_e g^d{}_f = g^b{}_d g^a{}_c R^{cd}{}_{ef} \qquad \Lambda^\mu{}_\nu \Lambda^\lambda{}_\rho \eta^{\nu\rho} = \eta^{\mu\lambda} \tag{28}$$

where $R = \mathrm{e}^{\mathrm{i}r} = I + \mathrm{i}r$ and it is assumed that $g^a{}_b$ are of the form (10). The R-matrix satisfies the 'unitarity condition': $\mathcal{P}R\mathcal{P} = \mathrm{e}^{\mathrm{i}\mathcal{P}r\mathcal{P}} = \mathrm{e}^{-\mathrm{i}r} = R^{-1}$ (here $\mathcal{P}$ is the permutation in the tensor product), but R does not satisfy the Yang–Baxter equation (because r does not satisfy the *classical* Yang–Baxter equation, cf (8)). The following result tells that the Yang–Baxter equation also does not hold for other possible choices of the R-matrix.

Proposition 1. A matrix R satisfies the first equation in (28) if and only if

$$R = t\mathrm{e}^{\mathrm{i}r} + s\mathcal{P}$$

where t and s are some numbers. R satisfies the Yang–Baxter equation only in the trivial case $t = 0$.

Proof. The first part of the proposition follows by a straightforward (lengthy) calculation, taking into account (10) and the defining relations for $\mathcal{A}$. Checking the Yang–Baxter equation

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

for R with $t \neq 0$, we can assume $t = 1$ and $R = I + \mathrm{i}r + s\mathcal{P}$. Denoting $e_\mu \otimes e_\nu \otimes e_\rho$ concisely by $|\mu\nu\rho\rangle$, we have

$$R_{12}R_{13}R_{23}|012\rangle = |012\rangle + s(|021\rangle + |102\rangle + |210\rangle) + s^2(2|120\rangle + |201\rangle) + s^3|210\rangle$$

$$R_{23}R_{13}R_{12}|012\rangle = |012\rangle + s(|021\rangle + |102\rangle + |210\rangle) + s^2(|120\rangle + 2|201\rangle) + s^3|210\rangle$$

hence the Yang–Baxter equation would imply $s = 0$. To obtain the above equalities, note that $r|\mu\nu\rho\rangle = 0$ for (μ, ν, ρ) being any permutation of $(0, 1, 2)$ (use (7), (6)). □

Let $\mathcal{B}$ denote the universal *-algebra with unity, generated by self-adjoint elements x^μ $(\mu = 0, 1, 2, 3)$, subject to the relations

$$[x^k, x^j] = 0 \qquad [x^k, x^0] = \mathrm{i}hx^k$$

$(j, k = 1, 2, 3)$. It is easily seen that the usual equation

$$\Delta_{\mathcal{B}}(x^\mu) = \Lambda^\mu{}_\nu \otimes x^\nu + v^\mu \otimes I$$

defines a unital *-homomorphism from $\mathcal{B}$ to $\mathcal{A} \otimes \mathcal{B}$. This represents the action of the quantum Poincaré group on the quantum Minkowski space associated with the algebra $\mathcal{B}$.

4. The κ-Poincaré algebra and its quasi-classical limit

In [10, 11], a quantized universal enveloping algebra in the sense of Drinfeld [1], describing a deformation of the Poincaré Lie algebra has been introduced in the form

$$[M_i, M_j] = \mathrm{i}\varepsilon_{ijk}M_k \qquad [P_\mu, P_\nu] = 0$$

$$[L_i, M_j] = \mathrm{i}\varepsilon_{ijk}L_k$$

$$[M_i, P_j] = \mathrm{i}\varepsilon_{ijk}P_k \qquad [M_i, P_0] = 0$$

$$[L_i, P_j] = \mathrm{i}\kappa\delta_{ij}\sinh\frac{P_0}{\kappa} \qquad [L_i, P_0] = \mathrm{i}P_i$$

$$[L_i, L_j] = -\mathrm{i}\varepsilon_{ijk}\left(M_k\cosh\frac{P_0}{\kappa} - \frac{1}{4\kappa^2}P_k(P\cdot M)\right).$$

Here $h := 1/\kappa$ should be interpreted as the deformation parameter and M_i, L_i, $i = 1, 2, 3$ and P_μ, $\mu = 0, 1, 2, 3$ are (as in section 2) the standard generators: rotational momenta, boosts and translational momenta. These quantities are self-adjoint, $M_i^* = M_i$, $L_i^* = L_i$ and $P_\mu^* = P_\mu$. The comultiplication is defined as

$$\Delta(M_i) = M_i \otimes 1 + 1 \otimes M_i$$

$$\Delta(L_i) = L_i \otimes \exp\left(\frac{P_0}{2\kappa}\right) + \exp\left(-\frac{P_0}{2\kappa}\right) \otimes L_i + \frac{1}{2\kappa}\varepsilon_{ijk}\left(P_j \otimes M_k \exp\left(\frac{P_0}{2\kappa}\right) + \exp\left(-\frac{P_0}{2\kappa}\right) M_j \otimes P_k\right)$$

$$\Delta(P_i) = P_i \otimes \exp\left(\frac{P_0}{2\kappa}\right) + \exp\left(-\frac{P_0}{2\kappa}\right) \otimes P_i \qquad \Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0.$$

Let us denote this quantized universal enveloping algebra by $\mathcal{U}_h(\mathfrak{g})$, where $\mathfrak{g}$ stands for the Poincaré Lie algebra. The quasi-classical limit of $\mathcal{U}_h(\mathfrak{g})$ is the Lie bialgebra $(\mathfrak{g}, \delta)$, where the cocommutator $\delta: \mathfrak{g} \to \bigwedge^2 \mathfrak{g}$ (obtained as $(\Delta - \sigma \circ \Delta) \bmod h^2$, σ is the permutation in the tensor product) is given by

$$\delta(M_i) = 0 \tag{29}$$

$$\delta(L_i) = hL_i \wedge P_0 + \tfrac{1}{2} h\varepsilon_{ijk} P_j \wedge M_k \tag{30}$$

$$\delta(P_i) = hP_i \wedge P_0 \qquad \delta(P_0) = 0. \tag{31}$$

It is easy to see that δ is a coboundary, i.e. there exists an element of $r \in \bigwedge^2 \mathfrak{g}$ such that $\delta := \partial r$ (that means that $\delta(X) = \mathrm{ad}_X r$ for $X \in \mathfrak{g}$). Equation (7) gives a solution to this problem.

5. Poisson dual of the Poisson Poincaré group

If we rescale the generators of the κ-Poincaré algebra

$$M_i \mapsto \frac{1}{\lambda} M_i \qquad L_i \mapsto \frac{1}{\lambda} L_i$$

(P_μ unchanged), we obtain the following commutation relations

$$[M_i, M_j] = \mathrm{i}\lambda\varepsilon_{ijk} M_k \qquad [P_\mu, P_\nu] = 0$$

$$[L_i, M_j] = \mathrm{i}\lambda\varepsilon_{ijk} L_k$$

$$[M_i, P_j] = \mathrm{i}\lambda\varepsilon_{ijk} P_k \qquad [M_i, P_0] = 0$$

$$[L_i, P_j] = \mathrm{i}\lambda\kappa\delta_{ij} \sinh\frac{P_0}{\kappa} \qquad [L_i, P_0] = \mathrm{i}\lambda P_i$$

$$[L_i, L_j] = -\mathrm{i}\lambda\varepsilon_{ijk}\left(M_k \cosh\frac{P_0}{\kappa} - \frac{1}{4\kappa^2} P_k (P \cdot M)\right)$$

with the equations for comultiplication unchanged. In this way we have embedded the κ-Poincaré algebra into a one-parameter family of Hopf *-algebras (the difficulty with non-polynomial functions of P_0 can be avoided in a standard manner by replacing P_0 by $K^{\pm} := \exp(\pm P_0/2\kappa))$. We interpret these Hopf *-algebras (and in particular the κ-Poincaré algebra) as the algebras of *functions* on some quantum groups. The quantum group corresponding to $\lambda = 0$ turns out to be a classical group (because the algebra of functions becomes commutative for $\lambda = 0$). Let us denote this group by G^*. It is known that linearizing the defining relations at the value of the parameter corresponding to the classical group yields a Poisson structure on this group. The group equipped with this structure then becomes a Poisson group. In our case, the Poisson structure is given simply by

$$
\begin{aligned}
&\{M_i, M_j\} = \varepsilon_{ijk} M_k && \{P_\mu, P_\nu\} = 0 \\
&\{L_i, M_j\} = \varepsilon_{ijk} L_k && \\
&\{M_i, P_j\} = \varepsilon_{ijk} P_k && \{M_i, P_0\} = 0 \\
&\{L_i, P_j\} = \kappa \delta_{ij} \sinh \frac{P_0}{\kappa} && \{L_i, P_0\} = P_i \\
&\{L_i, L_j\} = -\varepsilon_{ijk}\left(M_k \cosh \frac{P_0}{\kappa} - \frac{1}{4\kappa^2} P_k (P \cdot M)\right).
\end{aligned}
$$

One can of course check directly that the group G^*, whose multiplication law is defined by comultiplication equations from the previous section, is indeed a Poisson group with respect to the above Poisson structure. It is easy to see that the Poisson group G^* is exactly the dual Poisson group of the Poisson Poincaré group presented in section 2. To this end it is sufficient to show that the tangent Lie bialgebra of G^* is dual to the tangent Lie bialgebra of the Poisson Poincaré group. This is indeed so, because, on one hand, the linearization of the above Poisson structure at the group unit leads to linear Poisson brackets on $\mathfrak{g}^*$ which coincide (dually) with equations (1)–(5), and on the other hand, the Lie bracket on $\mathfrak{g}^*$ computed from the group law in G^*, has as the dual map the cobracket given in equations (29)–(31).

We conclude that the presentation of our Poisson Poincaré group and its (Poisson) dual looks exactly the same as the presentation of the quantum Poincaré group described in section 3 and the κ-Poincaré algebra, except that the Poisson brackets are replaced by commutators (divided by the imaginary unit). This suggests that the duality should also hold on the quantum level.

Finally, let us note, that the κ-Poincaré algebra is obtained from the Poisson dual to our Poisson Poincaré group by a simple quantization procedure: replacing Poisson brackets by commutators.

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